Quantum stirring as a probe of superfluidlike behavior in interacting one-dimensional Bose gases

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We propose quantum stirring with a laser beam as a probe of superfluidlike behavior for a strongly interacting one-dimensional Bose gas confined to a ring. Within the Luttinger liquid theory framework, we calculate the fraction of stirred particles per period as a function of the stirring velocity, the interaction strength, and the coupling between the stirring beam and the bosons. We show that the stirred fraction is never zero due to the presence of strong quantum fluctuations in one dimension, implying imperfect superfluid behavior under transport. Some experimental issues on quantum stirring in ring-trapped condensates are discussed.

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Progress in the ability to manipulate low-dimensional ultracold atomic gases has stimulated the interest in fundamental properties of one-dimensional (1D) Bose liquids.[1–3] A Bose-Einstein condensate (BEC) of an atomic gas is known to exhibit superfluidity. Experiments have confirmed the superfluid behavior by demonstrating a critical velocity below which a laser beam could be moved through the gas without causing excitations,[4,5] and an irrotational flow through the creation of vortices[6] and vortex lattices[7] in both rotating and nonrotating traps. For a Bose-Einstein condensate in a toroidal trap the observation of a persistent flow has also been reported.[8]

Parametric pumping offers another way of inducing particle transfer without creating excitations. In pumping, periodic (ac) perturbations of the system yield a dc current. Indeed, this current may be entirely adiabatic as long as the external perturbations are slow enough such that the system always remains in the instantaneous ground state. The number of particles transferred in each cycle is then independent of the pumping period T and the integral of the current over a period is quantized for a clean infinite periodic system with filled bands.[9,10] Up to now, spectacular precision of quantization of the pumped current has been achieved in experiments with nanoelectronics devices.[11]

Quantum pumping is intimately connected to quantum stirring. Quantum stirring is accomplished by the cyclical variation in one system parameter, while preserving the characteristic of a pump, i.e., the orientation of the particle flow is fixed. Quantum stirring has been exploited to elucidate the nature of the critical velocity in superfluid liquids[4,5,12] or the character of the fluid flow.[13]

We focus here on stirring a 1D-interacting Bose gas with a laser beam in the regime where interaction effects are especially strong, and we propose the fraction of stirred particles as a probe of superfluidlike behavior. Although for a homogeneous 1D Bose gas the superfluid fraction, defined as the response to twisted boundary conditions and estimated from ground-state quantities (see, e.g., Ref. 18), is always independent of the interaction strength, it is a relevant question to ask whether the *out of equilibrium* behavior of a 1D Bose gas as a probe is closer to the behavior expected for a superfluid (e.g., frictionless flow below a certain velocity threshold) or rather to a normal fluid (e.g., flow with drag). The study of the stirred fraction gives a measure of the degree of superfluidity of the fluid, i.e., a small stirred fraction corresponds to superfluidlike behavior and a unity-stirred fraction corresponds to normal-like behavior. Its measure is complementary to the onset of a drag force as a manifestation of superfluidlike behavior.[14,15]

We consider N bosons of mass m confined onto a 1D ring of circumference L, with contact interactions $v(x-x')=g\delta(x-x')$ at zero temperature. The long-wavelength behavior of this system at distances larger than the cut-off length $\alpha=1/\rho_0=L/N$ is described by the Luttinger liquid Hamiltonian in terms of the density and phase fluctuation modes of the bosonic field[16–18]

$$H_0=\frac{\hbar}{2\pi}\int dx\left\{\frac{v_s}{K}[\nabla\phi(x)]^2+v_sK[\nabla\theta(x)]^2\right\},\qquad(1)$$

where the field $\phi(x)$ is related to the particle density according to

$$\rho(x)=\left[\rho_0-\frac{1}{\pi}\nabla\phi(x)\right]\sum_{p=-\infty}^{\infty}e^{i2p[\pi\rho_0x-\phi(x)]},\qquad(2)$$

the field $\theta(x)$ corresponds to the phase of the superfluid, and we have $[\phi(x),\nabla\theta(x')]=i\delta(x-x')$. In the case of repulsive contact interaction between bosons, the Luttinger parameters v_s and K used in Eq. (1) are obtained (Ref. 18) by $v_sK=\frac{\pi\hbar\rho_0}{m}$, as follows from Galilean invariance, and $\frac{v_s}{K}=\frac{g}{\hbar\pi}$ in the weak-coupling limit, while $\frac{v_s}{K}=\frac{\pi\rho_0}{\hbar m}(1-\frac{8\rho_0\hbar^2}{mg})$ in the strong-coupling limit. When the interaction goes to zero, K goes to infinity, while $K=1$ for infinitely strong hardcore interactions [Tonks-Girardeau (TG) limit], where the problem can be solved by mapping onto a gas of noninteracting fermions.[19] In this regime $2\pi\rho_0\to 2k_F$, with k_F being the Fermi wave vector of the corresponding mapped spinless fermions. The long-wavelength properties of 1D dipolar gases are also described by Eq. (1) with $K<1$.[20]

We describe next the effect of a barrier moving with velocity V through the fluid by introducing the time-dependent potential $U(x,t)=U_0\delta(x-Vt)$. In an experiment this could be realized, e.g., by stirring the gas by a blue-detuned laser

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beam. The Hamiltonian acquires an explicitly time-dependent term which couples to the density:

$$\delta H(t) = \int dx U(x,t)\rho(x). \tag{3}$$

Using Eq. (2) for the density and keeping only the lowest most relevant harmonics we may rewrite Eq. (3) as

$$\delta H(t) = U_0\left\{\rho_0 - \frac{1}{\pi}\nabla\phi(Vt) + 2\rho_0\cos[2\pi\rho_0 Vt - 2\phi(Vt)]\right\}. \tag{4}$$

The term proportional to $\nabla\phi$ is analogous to a slowly varying chemical potential and can be absorbed in H by a redefinition of the field ϕ and $\phi \to \phi - (K/v_s)\int^x dx' U(x')$, while the last leading term in Eq. (4) represents scattering of the bosons off the barrier with momentum close to $\pm 2\pi\rho_0$. In the TG (Ref. 19) limit it describes the backscattering of right movers into left movers, i.e., processes with momentum close to $\pm 2k_F$. During its motion the barrier drags along a part of the bosons. We are interested in the *stirred fraction* $N_{\rm stir}/N$ i.e., the fraction of particles transported per period $T=L/V$ by the moving barrier, and related to the particle current as $N_{\rm stir}=\frac{1}{2\pi}\int_0^T dt I(t)$. If the barrier height is infinitely large, the fraction of stirred particles per period is quantized, i.e., $N_{\rm stir}/N=1$, independently of the interaction strength. If the barrier height is finite, the stirred fraction is in general smaller than one and we show that it is related to the degree of correlations in the system. We analyze perturbatively the regimes of weak and large barriers for arbitrary interaction strength and treat exactly the Tonks-Girardeau regime.

Weak barrier. In the weak-barrier limit we perform a perturbative analysis of the current generated by the stirring Hamiltonian δH. As customary in Luttinger liquid formalism we introduce the particle density of right (left) movers related to the fields $\theta(x)$ and $\phi(x)$ as $\rho_{R(L)} \simeq \frac{\rho_0}{2} \pm [\nabla\theta(x) \mp \nabla\phi(x)]$. The particle current at low energy is $J(x) \sim \nabla\theta(x)$; since it involves the difference in the number of right and left movers, the term proportional to $\nabla\phi$ in Eq. (4), which does not distinguish between left and right movers, plays no role in generating the particle current. On the contrary, the third backscattering term in Eq. (4) can lead to the generation of a current which we define of *backscattering*, I_b. In fact, addition of the moving-barrier potential breaks the continuous chiral symmetry[21] violating the conservation of the axial charge $N_R - N_L$, where $N_{R(L)}=\int dx \rho_{R(L)}(x)$. In the lowest-order perturbation theory the backscattering current is given by $I_b^0=\frac{i}{\hbar}[N_L,\delta H] = -\frac{i}{\hbar}[N_R,\delta H]$. In our specific case, by using the bosonized expression of the density operators and the stirring Hamiltonian, the resulting backscattering current operator is $I_b^0 = i\gamma(t)\tilde{n}(t) - \text{H.c.}$, where $\gamma(t)=U_0 e^{i2\pi\rho_0 Vt}$ and $\tilde{n} \sim \rho_0 e^{i2\phi(Vt)}$, and it is characterized by the backscattering frequency $\omega_b = 2\pi\rho_0 V$. Linear-response theory yields the backscattering current to second order in the barrier strength U_0 as

$$I_b \approx i\int_{-\infty}^{t} dt' \langle [I_b^0(t), \delta H(t')] \rangle_{H_0}, \tag{5}$$

and turns out to be related to the Fourier transform of the Green's function of the backscattering operator $e^{i2\phi(Vt)}$ at the characteristic frequency ω_b. In the thermodynamic limit $N, L \to \infty$ with $\rho_0=N/L$ constant and for small stirring velocity, the resulting backscattering current is given by

$$I_b \approx \frac{(2\pi)^{2K-1}}{\Gamma(2K)}\frac{U_0^2}{(\hbar v_s)^2}\left(\frac{V}{v_s}\right)^{2K-2} 2\pi\rho_0 V, \tag{6}$$

with Γ being the Euler gamma function. The fraction of stirred particles is readily obtained from the backscattering current according to $N_{\rm stir}/N = I_b/\omega_b$. In the Tonks-Girardeau limit $K \to 1$, Eq. (6) yields $N_{\rm stir}/N \propto (U_0/\hbar v_s)^2$, i.e., the result is independent of the frequency ω_b and hence adiabatic.[9,22] In the small ω_b limit this result is in agreement with the exact calculation of the fraction of stirred particles as shown below. Note that as the Luttinger liquid theory is an effective low-energy model, it describes correctly the system at frequencies $\omega_b < 2\pi v_s/\alpha$, hence expression (6) is valid only if $V < v_s$, and cannot treat the supersonic regime. By recalling that the power-law dependence in Eq. (6) originates from the excitation of sound waves in the quasi-one-dimensional geometry, we can also determine the smallest velocity for which Eq. (6) holds in the case of a ring of finite length. In this case no excitations are possible below the lowest velocity $V_{\rm low}=v_s/N \sim \pi\hbar/mL$ corresponding to the momentum of the lowest bosonic mode on the ring. The value of $V_{\rm low}$ found agrees with the one obtained by using a Gross-Pitaevskii approach for $K \gg 1$.[23] Thus as a main result we find that at the critical velocity $V_{\rm low}$ the fraction of stirred particles crosses from a power law to a constant (adiabatic regime). In this regime the fluid flow is steady.[13] Note also that the adiabatically stirred fraction decreases with decreasing interaction strength as $1/\Gamma(2K)$: when K grows, the dynamical response of the system becomes more superfluidlike, hence the interaction with the external barrier decreases and $N_{\rm stir}/N \to 0$.

We would also like to stress that in the interacting regime, no direct relationship between the superfluid fraction and the stirred fraction of particles can be found. In fact, the bulk superfluid fraction is defined in terms of v_s and K as mv_sK/π, while the stirred fraction of particles depends separately on K and v_s. Thus the latter does not serve as a direct measure of the superfluid fraction, but only as a probe of superfluidlike behavior under transport.

The results obtained above are consistent with a treatment based on the perturbative renormalization-group (RG) approach.[24] In this approach the scaling of the potential U_0 with frequency ω is obtained from the flow equation $dU_0/dz=(K-1)U_0$, where $dz=d\omega/\omega$. As a function of K, two regimes are distinguished. When $K>1$ the barrier is irrelevant: U_0 decreases as ω is decreased from v_s/α down to $\omega_b \sim V/\alpha$. For an infinite system, U_0 and hence $N_{\rm stir}/N$ scale to zero as $\omega_b \to 0$; for a finite system the RG procedure should be stopped when $\omega_b \sim v_s/L$, i.e., for $V \sim v_s/N$; this is the regime where we find a residual adiabatically stirred frac-

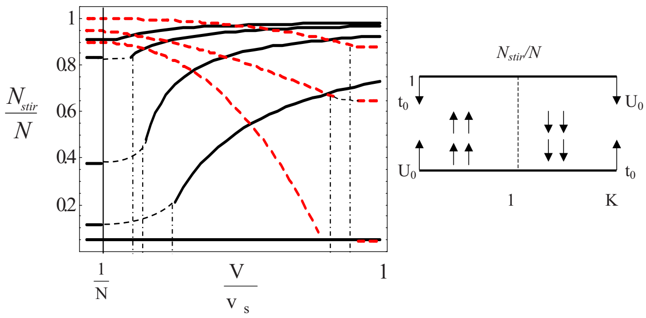

FIG. 1. (Color online) Left panel: fraction of stirred particles as a function of the stirring velocity (in units of v_s), obtained by matching Eqs. (6) and (7) at $V^{\star}_{U_0}=V^{\star}_{t_0}=V^{\star}$ through the RG considerations (see text). (Black) Continuous line: $N_{\rm stir}/N$ at decreasing barrier strengths $U_0/(\hbar v_s)$ from one (upper curve) to 0.2 (lower curve) for $K=2$; (red) dashed line: $N_{\rm stir}/N$ at increasing tunneling strength $t_0/(\rho_0\hbar v_s)$ from 0.2 (upper curve) to 0.6 (lower curve) for $K<1$, fixed at 0.5. The vertical dashed-dotted lines indicate $V^{\star}/v_s$. The vertical straight line indicates the critical velocity $V_{\rm low}$ below which the stirring is adiabatic. Right panel: summary of the RG flow for the barrier potential U_0 and tunneling strength t_0 (left and right edge arrows) and $N_{\rm stir}/N$ (arrows in the middle of the frame) at varying the interaction K approaching the adiabatic regime. The stirred fraction is analogous for neutral particles to the conductance for a 1D electron gas with a barrier (Ref. 24).

tion independent of V. For $K<1$, e.g., in the dipolar gas, U_0 and hence $N_{\rm stir}/N$ grow under RG, i.e., the barrier is a relevant perturbation. This is shown in the right panel of Fig. 1. Perturbation theory breaks down when $N_{\rm stir}/N\sim 1$, i.e., at the velocity $V^{\star}_{U_0}=v_s(U_0/\hbar v_s)^{1/1-K}$ and the RG flow must be stopped. The behavior beyond this breakdown point is described by an effective weak-link tunneling model.[24]

Weak-link limit. The large-barrier limit is equivalent to a ring cut at the position of the delta barrier, and we treat the residual tunneling t_0 between the two ends of the ring as a perturbation.[24] In this case the bosonized Hamiltonian corresponding to the hopping across the weak link can be obtained by a duality transformation,[24] $\phi\rightarrow\theta$ and is given by $\delta H\sim t_0\cos[2\pi\theta(Vt)]$. Its contribution to the particle current (*tunneling* current I_t) can be calculated in the linear-response regime and its explicit expression for an infinitely long ring is

$$I_t=\frac{(2\pi)^{2/K-1}t_0^2}{(\rho_0\hbar v_s)^2\Gamma(2/K)}\left(\frac{V}{v_s}\right)^{2/K-2}2\pi\rho_0 V. \qquad (7)$$

In the presence of tunneling the stirred fraction of particles is $N_{\rm stir}/N=1-I_t/\omega_b$, where I_t/ω_b is the fraction of tunneled particles, not stirred. In the hardcore limit ($K=1$) the stirred current will be again linear in the frequency of the stirring. We thus recover the adiabatic limit. Under the RG flow, the tunneling becomes relevant for interacting bosons with contact repulsion ($K>1$) therefore, upon decreasing the stirring velocity the effective tunneling strength increases, thereby decreasing the stirred particle fraction, again shown in the right panel of Fig. 1. Perturbation theory breaks down when the effective tunneling strength reaches unity and the RG flow must be stopped at $V^{\star}_{t_0}=v_s(t_0/\rho_0\hbar v_s)^{K/K-1}$, then the stirred fraction of particles is governed by the previous weak-barrier limit. The results for the dependence of the stirred fraction of particles on the velocity V are shown in Fig. 1. The results explicitly show a difference in the regime with $K>1$ (short-range interactions) and $K<1$ (dipolar interactions). In the latter case the stirred fraction decreases at increasing velocities, where the system shows a superfluid-like behavior. Since a dipolar gas is characterized by a quasicrystal-order phase at increasing density,[20] the result can be interpreted as an inefficiency of the stirring in creating an excitation in the ordered state.

Nonperturbative analysis. In the Tonks-Girardeau limit ($K=1$) a time-dependent Fermi-Bose (FB) mapping[19] is employed to generate exact solutions of the problem[25] and the current is calculated exactly. The time-dependent version of the FB mapping permits to write the exact many-body wave function of N impenetrable bosons on a ring as $\Psi_B(x_1,\ldots,x_N;t)=A(x_1,\ldots,x_N)\Psi_F(x_1,\ldots,x_N;t)$, where A is a unit antisymmetric function $A(x_1,\ldots,x_N)=\Pi_{1\le j<k\le N}{\rm sgn}(x_k-x_j)$, $\Psi_F(x_1,\ldots,x_N;t)=C\det_{i,j=1}^{N}\psi_i(x_j,t)$ is the wave function for any ideal Fermi gas, and $\psi_i(x_j,t)$ are the solutions of the one-body time-dependent Schroedinger equation in the external potential $U(x,t)$. Starting from the above many-body wave function, we evaluate the Tonks-Girardeau particle current density in terms of the one-body density matrix $\rho_1(x,y)=\int dx_2\cdots dx_N\Psi_B^*(x,\ldots,x_N;t)\Psi_B(y,\ldots,x_N;t)$ as $J(x)=-(\hbar/2mi)[\partial_r\rho_1(x+r/2,x-r/2)]_{r=0}$. Although $\rho_1(x,y)$ for a TG gas is very different from the one of a Fermi gas due to the presence of the mapping function A, we find that the latter has no effect on the current, which then coincides with the current of an ideal Fermi gas. In the adiabatic limit $V\le\pi\hbar/mL$ the particle current and the stirred fraction produced by the slow variation in the stirring potential can then be evaluated by following the adiabatic expansion of Thouless for an ideal Fermi gas,[9] i.e.,

$$N_s=\frac{i}{2\pi}\frac{\hbar^2}{mL}\int_0^{\tau}dt\sum_{\ell,j\neq 0}\frac{f_\ell(1-f_j)}{(\epsilon_\ell-\epsilon_j)}[\langle\psi_j|\dot{\psi}_\ell\rangle\langle\partial_x\psi_\ell|\psi_j\rangle+{\rm H.c.}], \qquad (8)$$

where $f_{\ell,j}$ is the fermionic probability occupation function of the state ℓ,j. In the case of a blue-detuned laser field piercing the ring at a position $x=0$ and modeled by the potential $U_0\delta(x)$, the appropriate orbitals $\psi_i(x_j,t)$ are the L-periodic free-particle energy eigenstates satisfying at $x=0$ the cusp condition. The complete orthonormal set of even-parity $\psi_n^{(+)}$ and odd-parity $\psi_n^{(-)}$ eigenstates are $\psi_n^{(+)}(x)=(e^{ik_nx}+e^{-ik_n(x-L)})/\mathcal{N}_n$ and $\psi_n^{(-)}(x)=\sqrt{\frac{2}{L}}\sin(2n\pi x/L)$, where k_n are obtained from the transcendental equation $k_n\tan(k_nL/2)=mU_0/\hbar^2$ (for $U_0\rightarrow\infty$ we have $k_n=\pi(2n+1)/L$, in agreement with Ref. 19) and $\mathcal{N}_n=\sqrt{2L[1+\sin(k_nL)/\,k_nL)]}$, with n running from one to ∞. The N-fermion ground state is obtained by inserting the lowest-N orbitals into the determinant above (Fermi sea), and using the exact orbitals $\psi_n^{(\pm)}$ as instantaneous ground state we obtain from Eq. (8)

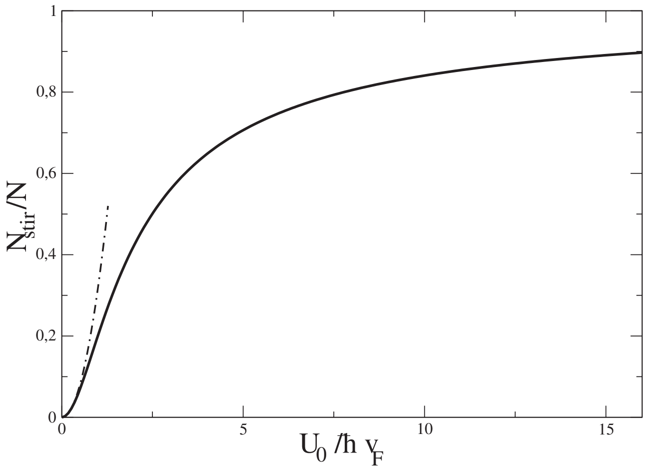

FIG. 2. Fraction of stirred particles for a Tonks-Girardeau gas (i.e., case $K=1$) as a function of the barrier strength $U_0/(\hbar v_s)$, in the adiabatic limit from Thouless expression Eq. (9) (solid line) and analytical small-U_0 behavior (dot-dashed line) as explained in the text.

$$N_s = 64\sum_{\ell,j}(f_\ell - f_j)\frac{\sin^2(k_jL/2)}{1+\sin(k_jL)/k_jL}\frac{(k_jL)^2 4\pi^2\ell^2}{[(k_jL)^2-4\pi^2\ell^2]^3}. \tag{9}$$

For a weak barrier by using the small-U_0 expression for k_n we obtain $N_s/N \simeq 0.32(U_0/\hbar v_s)^2$, which scales as the $K=1$ limit of the backscattered current in Eq. (6) because $v_s = \hbar k_F/m$ for $K=1$.[22] For an infinitely strong barrier using the $U_0 \to \infty$ limit of k_n it is straightforward to verify that the particle transport is quantized,[9] i.e., all the particles are dragged by the barrier and $N_{stir}/N=1$. This is shown in Fig. 2, where the stirred faction of particles is plotted as a function of the barrier strength.

Experimental issues on condensates in closed loop waveguide. A possible way of achieving experimentally an annular condensate with strong transverse confinement is to use a magnetic toroidal trap, as reported in Refs. 8, 26, and 27. Experimentally the stirring of hydrodynamic flow in a BEC by a blue-detuned laser beam, has been analyzed by calorimetric method[4] and phase contrast imaging[5] and recently in Ref. 13. The onset of a drag force has been shown by the asymmetry in the density profile, defined as the difference between the peak column density in front and behind the laser beam, as a function of the stirring velocity above a critical velocity. The space integral of the density asymmetry is analogous to the fraction of stirred particles calculated above. Recently, the persistent flow of Bose-condensed atoms in a toroidal trap has also been observed.[8] A variant to such experiment by the addition of a cyclic moving plug beam could be a valuable realization of the present proposal.

In conclusion, superfluid flow in a ring geometry raises interesting possibilities. With the use of a moving barrier acting as a quantum stirrer, the analog of quantization of particle transport for electron systems could be realized for a gas of atoms as an alternative probe of superfluidlike behavior.

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